M.Sc. (MATHEMATICS)

ASSIGNMENT

Session 2023-2025 (II-Semester)

&

Session 2022-2024 (IV-Semester)

CENTRE FOR DISTANCE AND ONLINE EDUCATION

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Programme: M.Sc. (Mathematics) Semester:-II

Important Instructions

(i) Attempt all questions from the each assignment given below. Each question carries marks mentioned in brace and the total marks are 15.

(ii) All questions are to be attempted in legible handwriting on plane white A-4 size paper and to be uploaded online to the Directorate of Distance Education for evaluation.

Nomenclature of Paper: Abstract Algebra

Paper Code: MAL-521 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Let V be a vector space over F and $T \in A(V)$. If $f(x) = a_0 + a_1 x + \ldots + a_{m-1}x_{m-1}$ $+ x_m$ is minimal polynomial of T over F and V is cyclic $F[x]-$ module, then prove that there exist a basis of V under which the matrix of T is companion matrix of $f(x)$. (5)

Q.2. Define similar transformation and prove that if subspace W of vector space is invariant under T, then T induces a linear transformation $\bar{T}$ on $\frac{V}{W}$ defined by $(v + W)\bar{T} = vT + W$. Further if T satisfies the polynomial $q(x)$ over F, then so does $\bar{T}$. (5)

Q.3. Define Nilpotent transformation with suitable example. Also prove that all the characteristic roots of a nilpotent transformation $T \in A(V)$ lies in F. (5)

ASSIGNMENT-II

Q.1. Show that if R is Noetherian ring with identity, then $R[x]$ is also Noetherian ring. (5)

Q.2. Let G be a finitely generated abelian group. Then prove that G can be decomposed as a direct sum of a finite number of cyclic groups C_i, i.e. $G = C_1 \oplus C_2 \oplus \ldots \oplus C_t$ where either all C_i's are infinite or for some j less then k, $C_1, C_2, \ldots C_j$ are of order $m_1, m_2, \ldots m_j$, respectively, with $m_1 | m_2 | \ldots | m_j$ and rest of C_i's are infinite. (5)

Q.3. Let M be an R-module. Then prove that the following conditions are equivalent. (5)

(i) M is semi-simple

(ii) M is direct sum of simple modules

(iii) Every submodule of M is direct summand of M.

Nomenclature of Paper: Measure & Integration Theory

Paper Code: MAL-522 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. State and prove Fatou's Lemma.

Q.2. State and prove bounded convergence theorem.

Q.3 Answer the following question:

(i) Differentiate between Lebesgue and Rieman integration.

(ii) What are Measurable function? Give example.

(iii) What are convex function?

(iv) Explain Lp- space with suitable example.

(v) What are function of bounded variation?

ASSIGNMENT-II

Q.1. State and prove Lebesgue theorem.

Q.2. State and prove Lusin theorem.

Q.3. Prove that if f and g be integrable over E. Then

(i) The function $(f + g)$ is integrable over E and

$$\int_E (f+g) = \int_E f + \int_E g$$

(ii) If $f \leq g$ a.e., then

$$\int_E \leq \int_E g$$

(iii) If A and B are disjoint measurable sets contained in E, then

$$\int_{AUB} = \int_A f + \int_B f$$

Nomenclature of Paper: Method of Applied Mathematics

Paper Code: MAL-523 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Find mean, variance and mean deviation about mean for the distribution having density function f(x) = $\frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$.

Q.2. Find moment generating function about origin and deduce the value of r^{th} moment for Chi-square distribution. Also obtain the values of mean and variance.

Q.3. Define Poisson distribution. If X is a Poisson variate such that

$$P(X = 2) = 9P(X = 4) + 90P(X = 6);$$

then find mean and standard deviation of the distribution.

ASSIGNMENT-II

Q.1. Find the Fourier cosine transform of f(t) = $\frac{1}{1+t^2}$. Hence derive Fourier sine transform of f(t = $\frac{1}{t(1+t^2)}$.

Q.2. Use the method of Fourier transforms to determine the displacement u(x; t) of an infinite string, given that the string is initially at rest and that the initial displacement is

f(x); ($-\infty < x < \infty$).

Q.3. Represent the vector $\vec{F} = z\,\hat{\imath} + 2x\,\hat{\jmath} + 3y\,\hat{k}$ in spherical co-ordinates (r; θ; ϕ).

Nomenclature of Paper: Ordinary Differential Equations-II

Paper Code: MAL-524 **Total Marks = 15 + 15**

ASSIGN MENT- I

Q1. Find the fundamental system of solutions of

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{bmatrix} t & 0 \\ 0 & 2t \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \text{in } [0,1]$$

Q2. State and prove Abel Liouville formula.

Q3. Obtain the solution ξ(t) of the initial value proble

$$X' = AX + B(t), \quad \xi(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B(t) = \begin{pmatrix} \sin at \\ \cos bt \end{pmatrix}$

ASSIGNMENT-II

Q1. Use calculus of variation to find the curve joining points (0, 0, 0) & (1, 2, 4) of shortest length. Also find the distance between these two points.

Q2. Determine the nature of the critical point (0, 0) of the system

$$\frac{dy}{dt} = 2x - 7y$$

$$\frac{dy}{dt} = 3x - 8y$$

and determine whether or not the point is stable.

Q3. Find the extremum (extremals) of the functional

$$I[y] = \int_1^2 \sqrt{\frac{1+y'^2}{x}}\,dx \quad \text{where} \quad y(1) = 0,\ y(2) = 1.$$

Nomenclature of Paper: Complex Analysis-II

Paper Code: MAL-525 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Prove that :

Let G be a region with

(i) The metric space Har(G) is complete.

(ii) If $\{u_n\}$ is a sequence in Har(G) such that $u_1 \leq u_2 \leq \ldots\ldots$ then either $u_n(z) \to \infty$ uniformly on compact subset of G or $\{u_n\}$ converges in Har(G) to a harmonic function.

Q.2. State and prove ' Riemann Mapping Theorem '.

Q.3. Prove that if $|z| \leq 1$ and $p \geq 0$ then $|1 - E_p(z)| \leq |z|^{p+1}$.

ASSIGNMENT-II

Q.1. State and prove ' Jensen Formula ' .

Q.2.(i) State Hadamard's Factorization theorem .

(ii) Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty}(1 - \frac{z^2}{n^2})$ by Hadamard's Factorization Theorem.

Q.3. (i). Define Genus and Exponential degree of an entire function.

(ii). Prove that the type σ of an entire function of finite order ρ is given by $\sigma = \overline{\lim_{r\to\infty}} \frac{\log M(r)}{r^\rho}$.

Nomenclature of Paper: Advanced Numerical Method

Paper Code: MAL-526 **Total Marks = 15 + 15**

ASSIGN MENT- I

Q.1. Obtain the cubic spline approximation valid in [3,4] for the function given in the tabular form

x	1	2	3	4
f(x)	3	10	29	65

under the natural spline condition $f''(1) = M(1) = 0$ and $f''(4) = M(4) = 0$.

Q.2. Approximate the value of the improper integral

$$I = \int_1^\infty x^{-3/2} \sin\frac{1}{x}\,dx.$$

Q.3. The function $f(x,y)$ is known as $(0,0) = -1, f(0,1) = 2, f(0,2) = 3, f(1,0) = 4, f(1,1) = 0, f(1,2) = 4, f(2,0) = 2, f(2,1) = -2, f(2,2) = 3$. For these values construct the Newton's bivariate polynomial. Also, find the approximate values of $f(1.25,0.75)$ and $f(1.0,1.5)$.

ASSIGNMENT-II

Q.1. Use Runge-Kutta method to solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$ correct to four decimal places. Initial values are $x = 0, y = 1$ and $y' = 0$.

Q.2. Solve the following system of linear equations by relaxation method taking $(0,0,0)$ as initial solution

$$27x_1 + 6x_2 - x_3 = 54$$
$$6x_1 + 15x_2 + 2x_3 = 72$$
$$x_1 + x_2 + 54x_3 = 110$$

Q3. Using a second order method with $h = 1/2$, find the solution of BVP

$$(1 + x^2)y'' + 2xy' - y = 1 + x^2$$
$$y(0) = 0,\ y'(1) = 1$$

Nomenclature of Paper: Computing Lab-Matlab

Paper Code: MAL-527 **Total Marks = 15 + 15**

ASSIGN MENT- I

Q.1. Write a program to calculate mean and median.

Q.2. Write a program to find the inverse of a matrix.

Q.3. Write a program to draw multiple graph on same plot.

ASSIGNMENT-II

Q.1. Write a program to operate arithmetic operators on vector.

Q.2. Write a program to find the multiplication of two matrices by using nested for loop.

Q.3. Write a program to operate element wise operations on matrices.

Programme: M.Sc. (Mathematics) Semester:-IV

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Nomenclature of Paper: Functional Analysis

Paper Code: MAL-641 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. State and prove Minkowski's Inequality.
Q.2. State ad prove Riesz-Representation Theorem for Hilbert spaces.
Q.3. Let M be a closed linear subspace of a Normed linear space N. If the norm of coset $x + M$ in the quotient space $\frac{N}{M}$ is defined by

$$\|x + M\| = \inf.\{ \|x + m\|; m\epsilon M\}.$$

Then $\frac{N}{M}$ is a normed linear space.

ASSIGNMENT-II

Q.1. State ad prove Riesz-Fisher Theorem.
Q.2. Prove that if a normed linear space X is reflexive, then X^* is also reflexive
Q.3. State and prove Open Mapping Theorem.

Nomenclature of Paper: Differential Geometry

Paper Code: MAL-642 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. (a) Established **Serret Frenet formulae** $\mathbf{t}' = k\,\mathbf{n}$, $\mathbf{n}' = \tau\mathbf{b} - k\mathbf{t}$, $\mathbf{b}' = -\tau\,\mathbf{n}$ where the symbols have their usual meaning.

(b) If C is a curve for which **b** varies differentially with arc length. Then to show that a necessary and sufficient condition that C is a plane curve is that $\tau = 0$ at all points.

Q.2.(a) For the curve $x = 3t, y = 3t^2, z = 2t^3$, show that any plane meets it in three points and deduce the equation to the osculating plane at $t = t_1$.

(b) Let C be a curve given by the equation $\mathbf{r} = (u, u^2, u^3)$, find the curvature and torsion of C at the point (0,0,0). Also, find the equation of its binormal line and normal plane at the point (1,1,1).

Q.3. Given the curve $\mathbf{r} = (e^{-u}\sin u,\ e^{-u}\cos u, e^{-u})$. Find at any point '$u$' of this curve

(i) Unit tangent vector **t**
(ii) The equation of tangent
(iii) The equation of normal plane
(iv) The curvature
(v) The unit principal normal vector **b**, and
(vi) The equation of the binormal.

ASSIGNMENT 1I

Q.1.(a) Find the principal curvatures and the lines of curvature on the right helicoids $x = u\cos\phi,\quad y = u\sin\phi,\quad z = c\phi$.

(b) Find the principal curvatures etc. on the surface generated by the binormals of a twisted curve.

Q.2.(a) Find the envelope of the plane $3xt^2 - 3yt + z = t^3$ and show that its edge of regression is the curve of the intersection of the surfaces $y^2 = zx$, $xy = z$.

(b) Find the envelope of the plane $(x/a)\cos\theta\sin\phi + (y/b)\sin\theta\sin\phi + (z/c)\cos\phi = 1$.

Q.3.(a) To prove that the envelope of a developable plane whose equation involves one parameter is a developable surface

(b) A necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normal along the curve is developable.

Nomenclature of Paper: Mechanics of Solid-II

Paper Code: MAL-643 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Derive the formulae for stresses in terms of two analytic functions, assuming plane strain conditions.

Q.2. Derive constitutive equation for a Maxwell material. Also discuss its creep and relaxation phases.

Q.3. Solve the problem of a long thick-walled tube in plane strain whose material is elastic in dilatation and Maxwell viscoelastic in distortion with internal pressure p and outer surface is in contact with a rigid body.

ASSIGNMENT-II

Q.1. Find torsional moment in the problem of torsion of an elliptic cylinder.

Q.2. Obtain the frequency equation for Rayleigh waves. Also show that these are non-dispressive and particle motion is elliptic retrograde.

Q.3. Discuss the problem of deflection of a central line of an elastic beam by transverse load.

Nomenclature of Paper: Integral Equation

Paper Code: MAL-644 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Find the integral equation corresponding to boundary value problem (B.V.P.)

$$y''(x) + \lambda y(x) = 0, \qquad y(0) = 0, y(1) = 1.$$

Q.2. State and prove Fredholm's Third Theorem.

Q.3. Solve the integral equation: $y(x) = x + \lambda \int_0^{\pi} \sin(x)\, sin(t)\, y(t) dt$.

ASSIGNMENT-II

Q.1. Find the resolvent kernel of Volterra Integral Equation with kernel $K(x, t) = \frac{\cosh t}{\sinh t}$.

Q.2. Transform the problem: $y''(x) + y = x$, $y(0) = 1$, $y'(1) = 0$ to Fredholm integral equation.

Q.3. State and prove Green's formula.

Nomenclature of Paper: Advanced Fluid Mechanics

Paper Code: MAL-645 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Derive Navier-Stokes's equation of motions in Cartesian coordinates.

Q.2. Define Reynold Number, Froude number, Mach number and Eckert number.

Q.3. Obtain the principal stresses and principal stress direction if the stress tensor at a point is given by

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

ASSIGNMENT-II

Q.1. Discuss the properties of boundary layer equations.
Q.2. Obtain the equation of motion of a gas.
Q.3. Determine the local frictional coefficient for flow over a flat plate, based on Karman integral equation.

Nomenclature of Paper: Computing Lab-3

Paper Code: MAP-648 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. What is use of multiline-environment, show by an example. How IEEE eqnarray – environment is used and what are the advantages. (5)
Q.2. Write syntax for the following

$$P_A(x) = \begin{cases} 1 & if\ \ x = 0 \\ 2 & if\ x = 1 \\ 4 & if\ x = \ -1 \end{cases} \quad (5)$$

Q.3. Discuss the commands that can be use to write multiple equations. (5)

ASSIGNMENT-II

Q.1. Write system for the following

$$\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right) \nabla.\vec{q} = 0$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + v\frac{\partial^2 T}{\partial y^2}\right). \quad (5)$$

Q.2. Write syntax for the following

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad (5)$$

Q.3. Construct following table using table environment of Latex

X	Y		Z
A	C_1	a	b
	C_2	c	d

B	C_3	e	f
		f	h