# M.Sc. (MATHEMATICS)

(Through Distance Education)

**ASSIGNMENTS**

**Session 2022-2024 (3rd Semester)**

**&**

**Session 2023-2025 (1st Semester)**

**DIRECTORATE OF DISTANCE EDUCATION**
**GURU JAMBHESHWAR UNIVERSITY OF SCIENCE & TECHNOLOGY**
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**Important Instructions for submission of Online Assignments**

i. Attempt all questions from the following assignments. Each question carries marks mentioned in brace.
ii. All questions are to be attempted in legible handwriting on plane white A-4 size paper along with front page and content table.
iii. Each page of the assignment carries Enrolment No.
iv. The Student will have to scan all pages of his/her handwritten assignment in PDF format size maximum 10 MB per assignment.
v. The students will have to upload assignments on student's portal.
vi. How to upload online Assignments check the Instructions for online submission of Assignment.

**Programme: M.Sc. (Mathematics) Semester:-I**

**Nomenclature of Paper: Algebra**
**Paper Code: MAL-511** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Define the Chief series and show that any two-invariant series for a given group have isomorphic refinements. (5)

**Q.2.** Prove that every normal series of a group $G$ is subnormal but the converse may not be true. (5)

**Q.3.** Write all the composition series for the Octic group. (5)

**ASSIGNMENT-II**

**Q.1.** Prove that regular pentagon is constructible. (5)

**Q.2.** Let characteristic of the field $F$ is $p(\neq 0)$. Then show that every algebraic extension $K$ of $F$ is separable if and only if the mapping $\sigma: F \to F$ given by $\sigma(a) = a^p$ is an automorphism of $F$. (5)

**Q.3.** Determine the Galois group of the following polynomials $x^2 + 1$, $x^3 - 2$ and $x^4 - 2$. (5)

**Nomenclature of Paper: Real Analysis**
**Paper Code: MAL-512** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Discuss Cauchy criterion for Uniform convergence of sequence and series of functions. (5)

**Q.2.** Evaluate $\int_0^1 x \, d\, x^2$ from definition of Riemann-Stieltjes Integral. (5)

**Q.3.** Show that the series: $(1 - x^2) + x(1 - x^2) + x^2(1 - x^2) + \ldots$.. is not Uniformly convergent on [0, 1].

## ASSIGNMENT-II

**Q.1.** A function *f* is integrable w.r.t. α on [a, b] if and only if for every ε > 0 there exist a partition P of [a, b] such that U(P, *f*, α) - L(P, *f*, α) < ε. (5)

**Q.2.** If $ax^2 + by^2 + cz^2 + 2fyz + 2gxz + 2hxy = k$ (constant) and $lx + my + nz = 0$, find maximum and minimum value of $x^2 + y^2 + z^2$. (5)

**Q.3.** Prove that the outer measure of an interval I is its length. (5)

**Nomenclature of Paper: Mechanics**

**Paper Code: MAL-513** **Total Marks = 15 + 15**

## ASSIGNMENT-I

**Q.1.** Define principal axes. Obtain expression for angular momentum and kinetic energy referred to principal axes.

**Q.2.** Find the directions of the principal axes at one of corners of a uniform rectangular plane lamina of mass M and side ‘‘ a and b``.

**Q.3.** Derive Lagrange`s equations for a planetary motion.

## ASSIGNMENT-II

**Q.1.** State and drive Whittaker`s equations.

**Q.2.** Define canonical transformation and show that the transformation

$P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}\frac{q}{p}$ is canonical.

**Q.3.** Find expression for potential at an internal point of a uniform solid sphere of radius `a`.

**Nomenclature of Paper: Ordinary Differential Equations-I**
**Paper Code: MAL-514** **Total Marks = 15 + 15**

## ASSIGNMENT-I

**Q.1.** Define the following along with an example:

(i) Linear Ordinary Differential Equation
(ii) Integral Equation
(iii) Ɛ-approximate solution
(iv) Sturm-Liouville Boundary Value Problem. (5)

**Q.2.** State & prove Gronwall’s Inequality theorem. (5)

**Q.3.** Apply the Runge-Kutta method to the initial value problem

$$\frac{dy}{dx} = 2x + y, \text{ given that } y(0) = 1.$$

Employ the method to approximate the value of the solution y at x = 0.2 and x = 0.4 using h = 0.2. Carry the intermediate calculations in each step to five figures after the decimal point and round off the final results of each step to four such places. Compare with the exact value. (5)

## ASSIGNMENT-II

**Q.1.** Solve IVP by Picard method

$$\frac{dx}{dt} = tx, \text{ given that } x(0) = 1. \qquad (5)$$

**Q.2.** let A(t) be a continuous n × n matrix defined on a closed and bounded interval I. Then the IVP

$$\frac{dx}{dt} = A(t)\,x(t), \text{ given that } x(t_0) = x_0, \quad t, t_0 \in I, \text{ has a unique solution on I.} \qquad (5)$$

**Q3.** Prove that the eigenvalues of a SLBVP are real.

**Nomenclature of Paper: Complex Analysis-I**
**Paper Code: MAL-515** **Total Marks = 15 + 15**

## ASSIGNMENT-I

**Q.1.** State and prove Cauchy Gourset theorem. (5)

**Q.2.** If a function $f(z)$ is analytic for all finite values of $z$ and is bounded, then $f(z)$ is constant. (5)

**Q.3.** Find the nature and location of the singularity of the following functions:

(i) $\frac{z-\sin z}{z^2}$ (ii) $\frac{1}{\cos z-\sin z}$ (iii) $(z+1)\sin\frac{1}{z-2}$ . (5)

## ASSIGNMENT-II

**Q.1.** State and prove Poisson`s Integral Formula. (5)

**Q.2.** Let $f(z)$ and $g(z)$ be analytic functions defined in the simply connected domain $D$ and $C$ be a simple close contour in $D$. If $|f(z)-g(z)| < |g(z)|$ holds for all $z$ on $C$, then $f(z)$ and $g(z)$ have the same number of zeros inside $C$. (5)

**Q.3** Find the Laurent Expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$. (5)

**Nomenclature of Paper: Programming with Fortran (Theory)**

**Paper Code: MAL-516** **Total Marks = 15 + 15**

## ASSIGNMENT-I

**Q.1.**Discuss the variable declaration,Syntax of a Fortran program, and list directed input/output statements. (5)

**Q.2.** Define and explain all Format specifications at the time of output statements. (5)

**Q.3.** Explain with flow charts the concept of nested-if in detail and discuss the Select Case. (5)

## ASSIGNMENT-II

**Q.1.** Describe with examples the Assignment statement, Arithmetic operators, Logical operators, and Relational operators. (5)

**Q.2.** Define Arrays and their features. Also, describe the String and operations of the string. (5)

**Q.3.** Define recursion and explain in brief the intrinsic functions. (5)

**Nomenclature of Paper: Programming with Fortran (Practical)**

**Paper Code: MAL-517** **Total Marks = 15 + 15**

## ASSIGNMENT- I

**Q.1**. Write a program to find the roots of a Quadratic Equation using arithmetic if statement.

**Q.2.** Write a program to check whether a given number is prime or not.

**Q.3.** Write a program for Bubble Sorting of an array.

## ASSIGNMENT-II

**Q.1.** Write a program to calculate factorial of a number N using Function.

**Q.2.** Write a program for fitting of a straight-line $y = mx + c$.

**Q.3.** Write a program to find transpose of a matrix.

# Program: M.Sc. (Mathematics) Semester:-3rd

**Important Instructions**

(i) Attempt both questions from each assignment given below. Each question carries marks mentioned in a brace and the total marks are 15 each.

(ii) All questions are to be attempted in legible handwriting on plane white A-4 size paper and to be submitted online to the Directorate of Distance Education for evaluation.

**Nomenclature of Paper: Topology**
**Paper Code: MAL-631** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Define the following along with an example:

(i) $T_1$-space
(ii) Lindelof space
(iii) Connected and Disconnected Set
(iv) Intersection of Product Sets
(v) Countably Compact (5)

**Q.2.** Let $X$ be a non-empty set. Let $c$ be an operator on $X$ satisfying following conditions:

(i) $A \subseteq c(A), \forall A \subseteq X.$
(ii) $c(\phi)=\phi$
(iii) $c(A \cup B) = c(A) \cup c(B), \forall A, B \subseteq X.$
(iv) $c(c(A)) = c(A), \forall A \subseteq X.$

Then, there exists a unique topology $\mathcal{T}$ on $X$ such that $c(A) = \bar{A}, \forall A \subseteq X,$ where $\bar{A}$ is $\mathcal{T}$-closure of $A$. (5)

**Q.3.** Let $(X, \tau)$ and $(Y, \tau')$ be two topological spaces. Let $f: X \to Y$ be a function. Let $x_0 \in X$, then $f$ is continuous at $x_0$ if and only if $A \subset X$, whenever $x_0 \in \bar{A}$, then $f(x_0) \in \overline{f(A)}$.

(5)

**ASSIGNMENT-II**

**Q.1.** A subspace of a real line is connected if and only if it is an interval. (5)

**Q.2.** Define the following along with an example:

(i) Limit Point
(ii) Completely regular space
(iii) Normal space
(iv) Neighbourhood
(v) Projection Map (5)

**Q.3.** Let $(X, \tau)$ be a topological space. Then the followings are equivalent.

a) $X$ is $T_1$ – space.
b) Every singleton is closed, i.e. $\{x\}$ is closed, $\forall\ x \in X$.
c) Given $x \in X$, $\{x\} = \cap\,\{H: H\ is\ a\ neighbourhood\ of\ x\}$. (5)

**Nomenclature of Paper: Partial Differential Equation**
**Paper Code: MAL-632** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Show that $u(x,t) = g(x - tb)$ is required solution of the initial value problem (5)

$$u_t + b.Du = 0 \text{ in } \mathbb{R}^n \times (0,\infty) \quad \text{and}$$
$$u = g \text{ on } \mathbb{R}^n \times \{t = 0\}$$

where $b \in \mathbb{R}^n$ and $g$ is the prescribed function.

**Q.2.** If $f$ is twice differentiable with compact support, then show that (5)

$$u(x) = \int_{\mathbb{R}^n} \emptyset(x-y) f(y)dy$$
$$= \begin{cases} -\frac{1}{2\pi}\int_{\mathbb{R}^n} \log|x-y| f(y)dy, & n = 2 \\ \frac{1}{n(n-2)\alpha(n)}\int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-2}} dy, & n \geq 3 \end{cases}$$

is a solution of Poisson's equation

$$\Delta\ = -u\,f \text{ in } \mathbb{R}^n$$

**Q.3.** Write short note on the followings: (5)

a) Kirchoff's formula,
d) Green's Function.

**ASSIGNMENT-II**

**Q.1.** Find the solution of heat equation (5)

$$u_t - \Delta u = 0 \quad \text{in } U \times (0,\infty),$$
$$u = 0 \text{ on } \partial U \times [0,\infty)$$
$$u = g \text{ on } U \times \{t = 0\}$$

where $g: U \to \mathbb{R}$ is given,

**Q.2.** Applying Fourier transform, solve the partial differential equation (5)

$$-\Delta u + u = f \text{ in } \mathbb{R}^n$$

where $f \in C^2(\mathbb{R}^n)$.

**Q.3.** Solve the Hamilton Jacobi equation (5)

$$u_t + H(Du) = 0 \text{ in } \mathbb{R}^n \times (0,\infty)$$

where $H$ is the Hamilton function.

**Nomenclature of Paper: Mechanics of Solid-I**
**Paper Code: MAL-633** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Define zero tensor and tensor of order zero. If $a_{ij}$ and $b_{mk}$ are second order tensors, then show that $a_{ij}b_{mj}$ is also a second order tensor and $a_{ij}b_{ij}$ is a scalar.
**Q.2.** Interpret geometrically the shear component of strain $e_{xy}$ .
**Q.3.** Show that the stress tensor $\tau_{ij}$ is symmetric.

## ASSIGNMENT-II

**Q.1.** Discuss Mohr`s diagram for the case when $\tau_2 = \tau_3$.

**Q.2.** The state of stress at a certain point of a body is given by $\begin{bmatrix} 200 & 400 & 300 \\ 400 & 0 & 0 \\ 300 & 0 & -100 \end{bmatrix}$.

Find the stress-vector on a plane passing through the point and parallel to the plane $x + 2y + 2z = 6$.

**Q.3.** Starting from stress-strain relations for an orthotropic medium, deduce the corresponding relations for an isotropic medium.

**Nomenclature of Paper: Advance Lab-II (MATLAB Programming & Applications)**
**Paper Code: MMP-634** **Total Marks = 15 + 15**

## ASSIGNMENT- I

**Q.1.** The hyperbolic sine for an argument $x$ is defined as $\sinh(x) = (e^x - e^{-x})/2$. Write an anonymous function to implement this. Use the function to make a plot of the function $\sinh(x)$ for $-6 \le x \le 6$.

**Q.2.** Write MATLAB code to find the curve of best fit of the type $y = be^{mx}$ to the following data

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| x | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
| y | 6.00 | 4.83 | 3.70 | 3.15 | 2.41 | 1.83 | 1.49 | 1.21 | 0.96 | 0.73 | 0.64 |

**Q.3.** Plot the function defined by $f(x) = x^3 - 12x^2 + 40.25x - 36.5$ on the domain $3 \le x \le 8$. Find the values and locations of the maxima and minima of the function.

## ASSIGNMENT-II

**Q.1.** Write Use MATLAB's built-in function ode45 with a suitable step size to solve the problem

$$\frac{dy}{dx} = \frac{x^3 - 2y}{x} \text{ for } 1 \le x \le 3 \text{ with } y = 4.2 \text{ at } x = 1.$$

**Q.2.** Solve the simultaneous equations $x - y = 2$ and $x^2 + y = 0$ using solve. Plot the corresponding functions, $y = x - 2$ and $y = -x^2$, on the same graph with $x$ range from $-5$ to 5.

**Q.3.** Write MATLAB codes based on Gauss Elimination for solving a system of linear equations.

**Nomenclature of Paper:** Fluid Mechanics
**Paper Code: MAL-**636 **Total Marks = 15 + 15**

## ASSIGNMENT-I

**Q.1** Derive the equation of continuity by vector approach for incompressible fluid. Interpret it physically. (5)

**Q.2** Show that if the velocity field $u(x, y) = \frac{B(x^2 - y^2)}{(x^2 + y^2)^2}$, $v(x, y) = \frac{2Bxy}{(x^2 + y^2)^2}$, $w(x, y) = 0$ satisfies the equation of motion for inviscid incompressible flow, then determine the pressure associated with this velocity field, B being a constant. (5)

**Q.3**. Find the velocity potential when a sphere is moving with constant velocity in a liquid which in otherwise at rest. (5)

**ASSIGNMENT-II**

**Q.1.** Show that the stream function and velocity potential for a two-dimensional irrotational motion satisfy Laplace's equation. (5)

**Q.2.** State and prove the Theorem of Blasius. (5)

**Q.3.** Find velocity potential, stream function, velocity components and complex potential due to a rectilinear vortex filament. (5)

**Nomenclature of Paper: Advance Discrete Mathematics**

**Paper Code: MAL-637** **Total Marks = 15 + 15**

**ASSIGNMENT-I**

**Q.1.** Prove that $p \leftrightarrow q \equiv (p \wedge q) \rightarrow (p \vee q)$ by using (a) truth table (b) algebra of propositions. (5)

**Q.2.** Let S = {1, 2, 3, 4}. We use the notation [12, 3, 4] ≡ [{1, 2},{3},{4}].
Three partitions of S follow: P1 = [12, 3, 4], P2 = [12, 34], P3 = [13, 2, 4], then find the other twelve partitions of S. (5)

**Q.3.** Let $L$ be a finite complemented distributive lattice. Then every element $a$ in $L$ is the join of a unique set of atoms. (5)

**ASSIGNMENT-II**

**Q.1.** Consider the following Boolean expression

$$E = x\,y' + x\,y\,z' + x'\,y\,z'$$

Then prove that $x\,z'$ is a prime implicant of E whereas x and $z'$ are not prime implicants of E. (5)

**Q.2.** Let $G$ be a finite graph with $n \geq 1$ vertices. Then the following are equivalent.
(i) $G$ is a tree
(ii) $G$ is a cycle-free and has $n - 1$ edges,
(iii) $G$ is connected and has $n - 1$ edges. (5)

**Q.3.** State and prove Warshall's **Algorithm**. (5)