Or

(a) Let E be a normal extension of F and let K be a subfield of E containing F. Show that E is a normal extension over K. Given an example to show that K need not be a normal extension of F. **8**

(b) Prove that finite extension of a finite field is separable. **4**

13. (a) Let F be a field of characteristic $\neq 2$. Let $x^2 - a \in F[x]$ be an irreducible polynomial over F. Then its Galois group is of order 2. **5**

(b) If $a > 0$ is contructible, then show that $\sqrt{a}$ is constructible. **6**

Or

(a) If $f(x) \in F[x]$ has r distinct roots in its plitting field E over F, then the Galois group G(E/F) of $f(x)$ is a subgroup of the symmetric group S_r. **6**

(b) Let E be the splitting field of $x^n - a \in F[x]$ then G(E/F) is a solvable group. **5**

Roll No. .... **Exam Code : J-19**

Subject Code—0351

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(First Semester)

MATHEMATICS

MAL-511

Algebra

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. If G is a cyclic group such that order (G) = $p_1 p_2 \ldots\ldots p_r$, p_i distinct primes, show that the number of distinct composition series of G is $r!$.

2. Show that a simple group is solvable if and only if it is cyclic.

3. Show that S_6 is not nilpotent.

4. Let $w = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$, and $u = \cos\frac{2\pi}{n}$, then show that $\left[\theta(w) : \theta(u)\right] = 2$.

5. Let k and k' be algebraic closures of a field F, then $k \simeq k'$ under an Isomorphism that is an identity of F.

6. Find the smallest normal extension (upto isomorphism) of $\theta\left(2^{1/4}, 3^{1/4}\right)$ in $\overline{\theta}$.

7. The prime field of a field F is either isomorphic to θ or to $\frac{z}{\langle p \rangle}$, p is prime.

8. The group $G\left(\theta(\alpha)/\theta\right)$, where $\alpha^5 = 1$ and $\alpha \neq 1$, is isomorphic to the cyclic group of order 4.

9. Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over θ.

10. Prove that the regular 17-gon is constructible with ruler and compass.

Section B

Note : Attempt all the questions.

11. (a) State and prove Jordon-Holder theorem. **8**

(b) Write down all the composition series for the quarternion group. **4**

Or

(a) State and prove Zassenhau's lemma. **8**

(b) A group G is nilpotent if and only if G has a normal series. **4**

12. (a) If E is an extension of F and $\mu \in$ E is algebraic over F, then F(u) is an algebraic extension of F. **6**

(b) Show that a finite field F of p^n elements has exactly one subfield with p^m elements for each divisor m of n. **6**