12. State and prove Extension theorem.

Or

Explain the behaviour of the solution of the IVP, depending upon initial function. **12**

13. State and prove the necessary and sufficient condition for a Pfaffian differential equation in three variables to be integrable. Hence solve the equation :

$$\left(2xyz + z^2\right)dx + x^2 zdy + \left(xz + 1\right)dz = 0$$

Or

State and prove Sturm-Liouville theorem. Verify the validity of the conclusion of S-L theorem for the characteristic functions of the following problem : **11**

$$\frac{d^2y}{dx^2} + \lambda y = 0;\ y\left(0\right) = 0,\ y\left(n/2\right) = 0$$

Roll No. .... **Exam Code : J-18**

Subject Code—0354

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(First Semester)

MATHEMATICS

MAL-514

Ordinary Differntial Equations-I

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Define an integral equation. Form an Integral equation corresponding to IVP :

$$y''(t) - 5y'(t) + 6y(t) = 0;\ y\left(0\right) = 0,\ y'(0) = -1$$

2. State Lipschitz condition. Given an IVP :

$$x'(t) = x^2 + \cos^2 t, \; x(0) = 0,$$

$$R = \left\{(t, x), 0 \le t \le a, |x| \le b, a \ge \frac{1}{2}, b > 0\right\}$$

Does it possess a unique solution ? Justify your answer.

3. Using Picard's method, find first four iterations of the IVP :

$$\frac{dy}{dx} = x + y^2, \; y(0) = 0$$

4. Using the method of undermined coefficient, solve the IVP :

$$\frac{dy}{dx} = x^3 + y^3, \; y(1) = 0$$

5. Describe maximal interval of existence in detail.

6. Transform the equation :

$$x'' - 2tx' + 2nx = 0$$

into an equivalent first order system.

7. If a solution of the differential equation $[p(t)u'(t)]' + q(t)u(t) = 0$ has an infinite number of zeros on [a, b], show that the solution is identically zero on [a, b].

8. Give an example of two differential equations through which you can verify Sturm Comparison Theorem.

9. Prove that eigen values of a Sturm Liouville Boundary Value Problem are real.

10. State and prove Gronwall's differential inequality.

Section B

Note : Attempt all the questions.

11. Define an $\in$-approximate solution. State and prove Cauchy Euler construction theorem.

Or

State and prove existence and Uniqueness theorem for a homogeneous system of n first order differential equation. **12**