

10. If $f(z)$ be an analytic function in the domain D , and let z_0 be a point in D . If $f'(z_0) \neq 0$, then $f(z)$ is conformal at z_0 .

Section B

Note : Attempt all the questions.

11. (a) State and prove Cauchy-Goursat theorem.
(b) State and prove Taylor's theorem. Also, find Taylor series for the function

$$f(z) = \frac{1+2z^3}{z+z^2}, \text{ valid in a neighbourhood of the point } z = i.$$

Or

- (a) State and prove Rouché's theorem.
(b) State and prove Poisson's formula.

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12. (a) Under the transformation $w = \log z$ find the image region of the annulus $a \leq |z| \leq b$.

Subject Code—0355

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(First Semester)

MATHEMATICS

MAL-515

Complex Analysis-I

Time : 3 Hours

Maximum Marks : 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. (a) If $f(z)$ is differentiable in a connected open set D and $|f(z)|$ is constant in D , then show that $f(z)$ is a constant in D .

- (b) If $f(z)$ is analytic on a simply connected region A then, for any two curves γ_1 and γ_2 joining two points z_0 and z_1 then prove that $\int_{\gamma_1} f(z)dz = \int_{\gamma_2} f(z)dz$.
2. Prove that $z = a$ be a zero of order k of an analytic function $f(z)$ iff $f(z) = (z - a)^k \phi(z)$ where $\phi(a) \neq 0, \phi(a) \neq \infty$.
3. (a) If $f(z)$ is continuous in a domain D and iff for every closed contour C in the domain D $\int_C f(z)dz = 0$ then prove that $f(z)$ is analytic within D .
- (b) Find $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$.
4. A function which is regular in all finite region of the complex plane, and is bounded, is identically equal to a constant.

5. State and prove maximum modulus principle theorem.
6. If the only singularities of an analytic function, including possibility the 'point at infinity' are poles, then prove that it is a rational function.
7. Classify the singularity of the following at the point mentioned against them :
- (a) $f(z) = \frac{1+2z^3}{z+z^2}$ at the point of infinity
- (b) $\frac{1}{1-e^z}$ at $z = 2\pi i$.
8. For the transformation $w = z^2$ find images of infinite strips between the lines $x = a, x = b$, and $x = -a, x = -b$.
9. State and prove Schwarz lemma.

Or

- (a) State and prove Laurent theorem. Also find Laurent series expansion, in power of $(z + 1)$, of the function

$$f(z) = \frac{z^2 + 1}{z(z^2 - 3z + 2)} \quad \text{in the region}$$

$$|z + 1| > 3.$$

- (b) If $f(z)$ has an isolated singularity at z_0 , then $f(z)$ has a pole at z_0 if and only if
- $$\lim_{z \rightarrow z_0} f(z) = \infty. \quad \mathbf{11}$$

- (b) State and prove Argument principle theorem.

Or

- (a) State and prove Mittag Leffler's expansion theorem.
- (b) Use method of contour integration prove that :

$$\int_0^{2\pi} \frac{1}{1 + a^2 - 2a \cos \theta} d\theta = \frac{2\pi}{1 - a^2}, \quad 0 < a < 1.$$

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- 13.** (a) State and prove sufficient condition for $w = f(z) = u + iv$ to be conformal mapping.

- (b) If $f(z)$ is analytic at z_0 with $f(z_0) \neq 0$ and $g(z)$ has a simple zero at z_0 then

$$\text{show that } \text{Res} \left[\frac{f(z)}{g(z)}; z_0 \right] = \frac{f(z_0)}{g'(z_0)}.$$