

12. (a) Prove that a family β of sets is a base for a topology for the set $X = \bigcup \{B : B \in \beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2$, $\exists$ a set $B \in \beta$ s.t. $x \in B \subseteq B_1 \cap B_2$. **6**

- (b) Show that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. **6**

Or

- (a) Prove that every sequentially compact top space is countably compact. **6**
- (b) Show that every sequentially compact metric space is compact. **6**

13. (a) Define first and second axiom space. Also show that every second axioms space is first axiom but converse need not be true. **6**
- (b) Prove that every compact Hausdorff space is regular. **5**

Subject Code—0356

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(Third Semester)

MATHEMATICS

MAL-631

Topology

Time : 3 Hours

Maximum Marks : 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. State and prove cofinite topology on a non-empty set.
2. Define interior of a set. Also show that :

$$\mathring{A} = \{y : y \in A \text{ and } y \notin d(X - A)\}$$

3. If f is a mapping of a top space X into another topological space, then prove that f is continuous on X iff

$$f(C(E)) \subseteq C^* f(E) \quad \forall E \subseteq X.$$

4. If C is a connected subset of a top space (X, T) which has a separation $X = A/B$, then show that either $C \subseteq A$ or $C \subseteq B$.
5. If E is a subset of a subspace (X^*, T^*) of a top space (X, T) , then show that E is T^* -compact iff it is T -compact.
6. Define locally compact space. Show that every compact top-space is locally compact but converse need not be true.
7. Show that a compact subset of a metric space is closed and bounded. Give an example to show that a compact subset of a top space need not be closed.
8. Define a separable space. Also show that every second countable space is separable.

9. Prove that a top space X is T_0 -space iff the closures of distinct points are distinct.
10. Define product topology and projections Π_x and Π_y of $X \times Y$ on X and Y respectively.

Section B

Note : Attempt all the questions.

11. (a) Prove that $\bar{A} = A \cup d(A) \quad \forall A \subseteq X$. **6**
- (b) Define topology in terms of Kurtowski closure operator. **6**

Or

- (a) Let T be the collection of subset of N consisting of ϕ and all subsets of N of the form $A_n = \{n, n+1, n+2, \dots\}$, $n \in N$. Show that T is a topology for N . Also find the derived set of $\{4, 13, 28, 35\}$. **6**
- (b) Show that a top space is connected iff it has no non-empty proper subset which is both open and closed. **6**

Or

- (a) Prove that a top-space X is completely normal iff every subspace of X is normal.

6

- (b) Show that the property of a space being normal is not a hereditary property. **5**

Or

- (a) Prove that a top-space X is completely normal iff every subspace of X is normal.

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- (b) Show that the property of a space being normal is not a hereditary property. **5**