Or

Show that there exists at most one weak unique solution of the initial value problem :

$$u_t - \mathrm{H}(\mathrm{D}u) = 0 \quad \text{in } \mathrm{R}^n \times (0, \infty)$$
$$u = g \quad \text{on } \mathrm{R}^n \times \{t = 0\}$$

where H is convex; $\lim\limits_{|p|\to\infty} \frac{H(p)}{|p|} = \infty$ and $g : \mathrm{R}^n \to \mathrm{R}$ is Lipschitz continuous.

13. Find the solution of :

$$u_{tt} - \Delta u = 0 \quad \text{in } \mathrm{R}^n \times (0, \infty)$$
$$u = g,\ u_t = h \quad \text{on } \mathrm{R}^n \times \{t = 0\}$$

for $n = 3$ and hence deduce it for $n = 2$.

Or

Define the Fourier transform of $u \in \mathrm{L}^1(\mathrm{R}^n) \cap \mathrm{L}^2(\mathrm{R}^n)$ then show that $\hat{u}, \breve{u} \in \mathrm{L}^2(\mathrm{R}^n)$ and

$$\|\hat{u}\|_{\mathrm{L}^2(\mathrm{R}^n)} = \|\breve{u}\|_{\mathrm{L}^2(\mathrm{R}^n)} = \|u\|_{\mathrm{L}^2(\mathrm{R}^n)}.$$

Roll No. .... **Exam Code : J-19**

Subject Code—0357

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(Third Semester)

MATHEMATICS

MAL-632

Partial Differential Equations

Time : 3 Hours *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Suppose $u = \mathrm{C}^2(\mathrm{R}^n) \cap \mathrm{C}(\bar{\mathrm{U}})$ is harmonic within U then $\max\limits_{\bar{\mathrm{U}}} u = \max\limits_{\partial \mathrm{U}} u$.

2. For Green's function $\mathrm{G}(x, y)$, show that $\mathrm{G}(x, y) = \mathrm{G}(y, x)$.

3. State and prove Euler Darboux equation.

4. Show that there exists at most one solution $u = C^2\left(\bar{U}_T\right)$:

$$u_t - \Delta u = f(x, y) \quad \text{in } R^n \times (0, \infty)$$
$$u = 0 \quad \text{on } R^n \times \{t = 0\}$$

5. Define domain of dependence of wave equation $u_{tt} - \Delta u = 0$ in $R^n \times (0, \infty)$. If $u \equiv u_t \equiv 0$ on $B(x, t_0) \times \{t = 0\}$ then $u = 0$ within the defined domain.

6. If $u, v \in L^1\left(R^n\right) \cap L^2\left(R^n\right)$ then show that $u = \left(\hat{u}\right)^{\vee}$.

7. Find the solution of the non-linear porous medium equation :

$$u_t - \Delta(u^\gamma) = 0 \quad \text{in } R^n \times (0, \infty)$$

where $u \geq 0$ and $\gamma > 1$ is a constant, using the similarity technique.

8. Obtain the exponential solution of Schrödinger's equation :

$$iu_t - \Delta u = 0 \quad \text{in } R^n \times (0, \infty)$$

and state the physical phenomenon.

9. Find the characteristic equations for the linear PDE :

$$F\left(Du, u, x\right) = \mathbf{b}(x).Du(x) + c(x)u(x) = 0 \quad \left(x \in U\right)$$

10. Solve using characteristics :

$$x_1 u_{x_1} + x_2 u_{x_2} = 2u; u\left(x_1, 1\right) = g\left(x_1\right)$$

Section B

Note : Attempt all the questions.

11. Solve, with the help of Green's function :

$\Delta u = 0$ in $B(0, 1)$ $\quad u = g$ on $\partial B(0, 1)$

And hence deduce it for $B(0, r)$.

Or

State and prove the mean value property for the equation $\Delta u = 0$. State the converse part also. **11**

12. Solving the homogeneous equation, find the solution of :

$$u_t - \Delta u = f(x, t) \quad \text{in } R^n \times (0, \infty)$$
$$u = 0,\ u_t = 0 \quad \text{on } R^n \times (t = 0\}$$

using Dhumel's principle.