(b) Define equal tensors and isotropic tensor. If a_{ik} and b_{mj} are second order tensors, then prove that $a_{ij}b_{ij}$ is a scalar. **6+6**

Or

Obtain Saint-Venant's equations of compatibility along its physical interpretation. Also explain physical interpretation of cubical dilation $v = e_{kk}$. **12**

12. (a) Derive the necessary and sufficient condition for an infinitesimal affine transformation to represent a rigid body motion.

(b) Derive the equation of equilibrium for a deformed elastic body. **6+6**

Or

Discuss Mohr's diagram for finding the maximum shearing stresses. **12**

Roll No. .... **Exam Code : J-19**

Subject Code—0358

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(Third Semester)

MATHEMATICS

MAL-633

Mechanics of Solids-I

Time : 3 *Hours* *Maximum Marks* : 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. If A_{ij} is a skew-symmetric second order tensor, prove :

$$(\delta_{ij}\delta_{lk} + \delta_{il}\delta_{jk})A_{ik} = 0$$

Also define symmetric and skew-symmetric tensors.

2. Prove that gradient of a scalar field is a first order tensor.

3. State and prove quotient law of tensors.

4. Refer the quadric of deformation to a set of principal axes and discuss the nature of deformation when the quadric is an ellipsoid and when it is hyperboloid.

5. Give geometrical interpretation of shear component of strain e_{13}.

6. Write a short note on stress quadric of Cauchy.

7. The state of stress at a point is given by :

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Show that the normal component of the stress-vector on a plane with normal in the direction (1, 1, 2) has unit magnitude. Also find the shear stress.

8. Show that if $\sigma = 0$, then $\lambda = 0$, $2\mu = E$, $3k = E$ and the generalized Hooke's law becomes :

$$\tau_{ij} = Ee_{ij} = \frac{1}{2}E(u_{i,j}+u_{j,i})$$

Further, show that :

$$\tau_{ij,ij} = \frac{1}{2}(\tau_{ii,jj}+\tau_{jj,ii})$$

9. Define Lame's constants and Poisson's ratio. Also explain physical significance of Poisson's ratio.

10. If $\vec{E} = \nabla\phi$ and $\tau_{ij} = E_iE_j - \frac{1}{2}\delta_{ij}E^2$, show that the equations of equilibrium are satisfied provided $\vec{F} = -\nabla\phi\left(\nabla^2\phi\right)$.

Section B

Note : Attempt all the questions.

11. (a) Prove the relation :

$$\in_{ijm}\in_{klm} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$$

13. State generalized Hooke's law. Derive its form for a medium with one plane of elastic symmetry. **11**

Or

Prove that :

$$\nabla^2\theta = -\left(\frac{1+\sigma}{1-\sigma}\right)\text{div}\,\vec{F}$$

where symbols have their usual meanings. **11**

13. State generalized Hooke's law. Derive its form for a medium with one plane of elastic symmetry. **11**

Or

Prove that :

$$\nabla^2\theta = -\left(\frac{1+\sigma}{1-\sigma}\right)\text{div}\,\vec{F}$$

where symbols have their usual meanings. **11**