12. (a) Prove that for every pair of elements a and b in Boolean algebra :

$$(a+b)+(a'*b')=1$$

and $(a+b)*(a'*b')=0$

(b) Prove that the following are equivalent in Boolean algebra :

(i) $a+b=b$

(ii) $a*b=a$

(iii) $a'+b=1$

(iv) $a*b'=0$

Or

(a) Explain AND, OR and NOT gates.

(b) Write a short note on Karnaugh map for case of two variables. **12**

13. (a) Define the following along with one example for each : matrix representation of graphs, weighted graphs, strong connectivity and directed tree.

(b) Prove that the graph $K_{3,3}$ is not planar.

Subject Code—0360

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(Third Semester)

MATHEMATICS

MAL-635

Advance Discrete Mathematics

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Explain tautology and contradiction. Check whether the proposition $(p \vee q) \vee \sim p$ is a tautology or not ?

2. Construct the truth tables for the following : $\sim(p \vee q) \vee (\sim p \vee q)$ and $\sim(p \vee q) \vee \sim(q \leftrightarrow p)$

3. What do you mean by propositional function ? Let A = {1, 2, 3, 4, 5}, determine the truth value of each of the following statement :

(i) $(\exists x \in A)\ (x + 3 = 10)$

(ii) $(\forall x \in A)\ (x + 3 \leq 7)$

(iii) $(\exists x \in A)\ (x + 3 < 5)$

(iv) $(\forall x \in A)\ (x + 3 < 10)$

4. Define partially ordered sets and prove that "less than or equal to" is a partial ordering on the set of positive integers.

5. Show that if L_1 and L_2 are distributive lattices then $L = L_1 \times L_2$ is also distributive.

6. Define a literal and fundamental product. Express the Boolean expression $E(x, y, z) = z(x' + y) + y'$ in its complete sum of products form.

7. Explain the following terms in graph theory : Directed graph; Indegree and outdegree of a vertex; Simple graph and cycle.

8. State and prove Euler's formula for planar graphs.

9. Define Spanning tree and Minimal spanning tree. Give an example for each.

10. Let G be a non-empty graph with at least two vertices, the graph G is bipartite if and only if it has no odd cycle.

Section B

Note : Attempt all the questions.

11. (a) Explain the basic logical connectives : Conjunction, disjunction and negation along with one example for each.

(b) What are the different types of quantifiers ? Using quanitifiers, prove that $\sqrt{2}$ is an irrational number.

Or

(a) State and prove associate laws in a lattice.

(b) Show that every chain is a distributive lattice. **12**

Or

(a) Write a short note on Dijkstra's algorithm.

(b) Prove that a connected graph G with n vertices is a tree iff it has exactly $(n-1)$ edges.

Or

(a) Write a short note on Dijkstra's algorithm.

(b) Prove that a connected graph G with n vertices is a tree iff it has exactly $(n-1)$ edges.