11. Let $A_F(V)$ be the space of all similar transformations. Then the elements S and T in $A_F(V)$ are similar iff they have the same elementary divisors.

Or

Let A be a 5×5 matrix with eigen value λ of multiplicity 5. Write all possible Jordan Canonical forms. **12**

12. If a linear transformation T on a vector V over a field F, has all its eigen values in F, then there exists a basis of V in which the matrix T is a triangular.

Or

State and prove Noether-Lasker theorem. **12**

13. Let $T \in Hom_F(V, V)$. Then there exists a basis of V such that the matrix of T relative to this basis is a direct sum of generalized Jordan blocks.

Or

Prove that every matrix is equivalent to a "diagonal" matrix (Smith normal form). **11**

Roll No. .... **Exam Code : J-19**

Subject Code—0361

M. Sc. EXAMINATION

(Second Semester)

(Main and Re-appear For Batch 2011 Onwards)

MATHEMATICS

MAL-521

Abstract Algebra

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Define similarity of linear transformations and prove that the relation of similarity is an equivalence relation.

2. There exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$ where V_1 is the subspace of V spanned by $V_1 = V$, $V_2 = VT$, ... $V_{n1} = vT^{n_1-1}$ and invariant under T.

3. Two nilpotent linear transformations are similar iff they have the same invariants.

4. Let $T \in A_F(V)$, the space of all similar linear transformations, and

$$p(x) = r_0 + r_1 x + ...r_{r-1}x^{r-1} + x^r$$

be the minimal polynomial of T over F. Further let V be a cyclic module. Show that the elements v, vT, ...vT^{r-1} forms a basis of V over F.

5. If R is PID, then for any integer n, any submodule of R^n is free of rank at most n.

6. Let every quotient module of an R-module M be finitely cogenerated, then every nonempty set S of submodules of M has a minimal element.

7. State and prove Hilbert basis theorem.

8. Obtain the Smith normal form and rank of the matrix over a PID R :

$$\begin{bmatrix} 0 & 2 & -1 \\ -3 & 8 & 3 \\ 2 & -4 & -1 \end{bmatrix}$$

where R = Z.

9. Find rational canonical forms of the following matrix over Q :

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

10. Let R be a left (or right) artinian ring with unity and no non-zero nilpotent ideals. Then R is also right and left artinian (noetherian).

Section B

Note : Attempt all the questions.