10. Define multiple correlation. Obtain coefficient of multiple correction $R_{3.12}$.

Section B

Note : Attempt all the questions.

11. (a) Solve, using Fourier transforms :

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t},\ t > 0$$

(b) Obtain Fourier cosine transform of :

$$f(t) = e^{-t^2/2}$$ **6+6**

Or

Prove that the spherical co-ordinate system (r, θ, ϕ) is orthogonal. Also express the vector :

$$\vec{A} = y\hat{i} - 2z\hat{j} + x\hat{k}$$

in spherical co-ordinates. **12**

12. (a) Obtain expression for velocity and acceleration for a particle with position vector $\vec{r}$ in cylindrical co-ordinates.

Roll No. .... **Exam Code : J-19**

Subject Code—0363

M. Sc. EXAMINATION

(For Batch 2011 Onwards)

(Main & Re-appear)

(Second Semester)

MATHEMATICS

MAL-523

Methods of Applied Mathematics

Time : 3 Hours *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Obtain Fourier sine transform of $\dfrac{1}{t\left(t^2 + b^2\right)}$.

2. Find the Fourier transform of :

$$f(t) = \begin{cases} 1-t^2; & |t| \le 1 \\ 0 \quad ; & |t| > 1 \end{cases}$$

Also state change of scale property on Fourier transforms.

3. A semi-infinite medium $x \ge 0$ is initially at temperature zero. At time $t = 0$, a constant temperature u_0 is applied and maintained at the face $x = 0$. Find the temperature at any point of the medium and at any time $t > 0$.

4. Obtain expression for Curl $\vec{F}$ in orthogonal curvilinear co-ordinates (u, v, w).

5. If u_1, u_2, u_3 are general co-ordinates, show that :

$$\frac{\partial \vec{r}}{\partial u_1}, \frac{\partial \vec{r}}{\partial u_2}, \frac{\partial \vec{r}}{\partial u_3}$$

and $\nabla u_1, \nabla u_2, \nabla u_3$ are reciprocal system of vectors.

6. Find mean and variance for the continuous probability distribution with :

$$f(x) = \frac{3}{4}x(2-x),\ 0 \le x \le 2$$

7. Define moment generating function. If X assumes the value "x" with probability :

$$P(X = x) = pq^{x-1},\ x = 1, 2, 3, ...$$

find its moment generating function and hence deduce the value of mean and variance.

8. In a Poisson distribution, frequency corresponding to 3 successes in $\frac{2}{3}$ times the frequency corresponding to 4 successes. Obtain the the mean and standard deviation of the distribution.

9. If X is distributed normally with mean "$\bar{x}$" and standard deviation "σ", prove that $U = \frac{1}{2}\left(\frac{X-\bar{x}}{\sigma}\right)^2$ is a Gamma variate with parameter $\frac{1}{2}$.

(b) If X has the discrete distribution :

$$f(x)=\begin{cases}\dfrac{1}{b-a}, & a\le x\le b\\ 0, & \text{otherwise}\end{cases}$$

show that $E(X)=\dfrac{a+b}{2}, \sigma^2=\dfrac{1}{12}(b-a)^2$

and $E\left[|X-\text{mean}|\right]=\dfrac{b-a}{4}$. **6+5**

(b) Prove that, for a normal distribution, the deviation about mean is equal to $\frac{4}{5}$ times the standards deviation. **6+6**

Or

(a) Show that the M.G.F. about mean of a Binomial distribution tends to that of Poisson distribution as $n\to\infty$ and $np = m$ is finite.

(b) Define exponential distribution. Obtain its M.G.F. Hence deduce the values of mean and variance. **6+6**

13. (a) State and prove weak law of large numbers.

(b) Define *t*-distribution and obtain its mean and variance. **6+5**

Or

(a) Prove that for a normal distribution, all odd order central moments are zero.