Or

Define periodic system of differential equations. State and prove representation theorem for the same. Also, derive variance of constant formula for non-homogeneous linear system.

12. Given that the roots of the characteristic equation of a linear autonomous system are real, unequal and of same sign. Prove that the critical point of the linear system will be a node. Discuss the stability of the critical point of the differential equation of motion $m\frac{d^2x}{dt^2}+\alpha\frac{dx}{dt}+kx=0$, where $m>0, \alpha \geq 0$ and $k>0$.

Or

Define limit cycles. Find the limit cycles of the system :

$$\frac{dx}{dt}=y+x\left(1-x^2-y^2\right)$$

$$\frac{dx}{dt}=-x+y\left(1-x^2-y^2\right)$$

Roll No. .... **Exam Code : J-19**

Subject Code—0364

M. Sc. EXAMINATION

(For Batch 2011 Onwards) (Main & Re-appear)

(Second Semester)

MATHEMATICS

MAL-524

Ordinary Differential Equations-II

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Define Wronskian for a homogeneous linear system. If the vector functions ϕ_1, ϕ_2, ..., ϕ_n are linearly dependent on I, then prove that Wronskian $\omega(\phi_1,.......,\phi_n)(t)=0$ for every $t \in$ I.

2. If Φ is a fundamental matrix of the system $x'(t) = A(t)x(t)$ and C is a constant non-singular matrix, then prove that ΦC is a fundamental matrix.

3. Explain the process of reduction of order of an *n*th order homogeneous linear differential equation.

4. Find the solution and the fundamental matrix of $x'(t) = Ax(t)$, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

5. Discuss classification of critical points of an autonomous system.

6. Determine the nature of critical point (0, 0) of the linear autonomous system :

$$\frac{dx}{dt} = 2x - 4y, \frac{dy}{dt} = 2x - 2y$$

and its stability.

7. Find the plane curve of shortest length joining two points $A(a_1, b_1)$ and $B(a_2, b_2)$.

8. State and prove fundamental Lemma of calculus of variations.

9. Find the differential equation of the lines of propagation of light in an optically non-homogeneous medium in which the speed of light is $v(x, y, z)$

10. Explain geodesic in context of variational problem with a suitable example.

Section B

Note : Attempt all the questions.

11. (i) State and prove Abel's-Liouville formula for a linear homogeneous system.

(ii) A solution matrix Φ of the matrix differential equation $x'(t) = A(t)x(t)$, $t \in I$ is a fundamental matrix iff det $\Phi(t) \neq 0$ for any $t \in I$.

Define index of a critical point for an autonomous system. **12**

13. Derive Euler's equation for functionals containing one dependent variable, its first order derivative and one independent variable. Find the curve with fixed boundary points such that its rotation about the axis of abscissa gives rise to a surface of revolution of minimum surface area.

Or

Define Isoperimetric problems. Solve the problem $I[y] = \int_{-a}^{a} y\, dx$ = maximum subject to the conditions :

$$y(-a) = y(a) = 0 \text{ and } J(y) = \int_{-a}^{a} \sqrt{1 + y'^2}\, dx = l$$

Find the geodesics on a right circular cylinder of radius a. **11**

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