Or

(a) If $n(r)$ denotes the number of zeros of the entire function $F(z)$ in the disc $|z| \le r$ and $F(0) \ne 0$ then

$$\int_0^R \frac{n(t)}{t} dt \le \log M(R) - \log|F(0)|.$$

(b) State and prove Hadamard's three circle theorem.

12. (a) State and prove Monodromy theorem.

(b) State and prove Great Picard theorem.

Or

(a) State and prove Schottky's theorem.

(b) State and prove Bloch's theorem.

13. State and prove Harnack's theorem.

Or

Define order of an entire function of finite order. Also, prove that the order ρ of an entire function is given by the formula

$$\rho = \lim_{r\to\infty} \frac{\log\log M(r)}{\log r} \left(r = |z|\right).$$

Roll No. .... **Exam Code : J-19**

Subject Code—0365-X

M. Sc. EXAMINATION

(Prior 2011 Re-appear)

(Second Semester)

MATHEMATICS

MAL-525

Complex Analysis-II

Time : 3 Hours *Maximum Marks :* 100

Section A

Note : Attempt any *Seven* questions. **7×7=49**

1. State and prove Montel theorem.

2. Define Zeta function and prove that for Re $s > 1$.

$$\Gamma(r)\zeta(s) = \int_0^\infty \frac{t^{s-1}}{e^{t-1}} dt$$

3. Define Gamma function. Also discuss the existence of the Euler constant involved in the Gamma function and prove that :

$$\Gamma(z) = \lim_{n\to\infty} \frac{n!n^z}{z(z+1)(z+2)....(z+n)}$$

4. State and prove Poisson-Jensen formula.

5. The exponent of convergence of the zeros of an entire function of finite order ρ_c is not greater than ρ.

6. Let :

$$|a_n| = r_n \quad (0 < r_1 \le r_2 \le r_3 .... \le r_n \le ....; r_n \to \infty)$$

then the exponent of convergence is given by $\frac{1}{\rho_c} = \lim_{n\to\infty} \frac{\log r_n}{\log n}$.

7. State and prove Weirestrass Factorization theorem.

8. Using Hadamard's factorization theorem show that :

$$\frac{\sin \pi\sqrt{z}}{\pi\sqrt{z}} = \prod_{n=1}^{n}\left(1 - \frac{z}{n^2}\right)$$

9. State and prove Montel-Caratheodory theorem.

10. State and prove little Picard's theorem.

Section B

Note : Attempt all the questions. **3×17=51**

11. (a) State and prove Riemann maping theorem.

(b) Let (X, d) be a compact metric space and let $\{g_n\}$ be a sequence of continuous function from X into **C** such that $\sum g_n(x)$ converges absolutely and uniformly for x in X. Then the product

$$f(x) = \prod_{n=1}^{\infty}(1 + g_n(x))$$ converges absolutely

and uniformly for x in X. Also there is an integer n_0 such that $f(x) = 0$ iff $g_n(x) = -1$ for some n, $1 \le n \le n_0$.