Or

(a) If $n(r)$ denotes the number of zeros of the entire function $F(z)$ in the disc $|z| \le r$ and $F(0) \neq 0$ then

$$\int_0^R \frac{n(t)}{t} dt \le \log M(R) - \log|F(0)|.$$

(b) State and prove Hadamard's three circle theorem. **12**

12. (a) State and prove Monodromy theorem.

(b) State and prove Great Picard theorem.

Or

(a) State and prove Schottky's theorem.

(b) State and prove Bloch's theorem. **12**

13. State and prove Harnack's theorem.

Or

Define order of an entire function of finite order. Also, prove that the order ρ of an entire function is given by the formula

$$\rho = \lim_{r \to \infty} \frac{\log \log M(r)}{\log r} \left(r = |z|\right).$$ **11**

Roll No. .... **Exam Code : J-19**

Subject Code—0365

M. Sc. EXAMINATION

(Main & Re-appear For Batch 2011 Onwards)

(Second Semester)

MATHEMATICS

MAL-525

Complex Analysis-II

Time : 3 Hours *Maximum Marks : 70*

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. State and prove Montel theorem.

2. Define Zeta function and prove that for Re $s > 1$.

$$\Gamma(r)\zeta(s) = \int_0^\infty \frac{t^{s-1}}{e^{t-1}} dt$$

3. Define Gamma function. Also discuss the existence of the Euler constant involved in the Gamma function and prove that :

$$\Gamma(z) = \lim_{n\to\infty} \frac{n!n^z}{z(z+1)(z+2)....(z+n)}$$

4. State and prove Poisson-Jensen formula.

5. The exponent of convergence of the zeros of an entire function of finite order ρ_c is not greater than ρ.

6. Let :

$$|a_n| = r_n \quad (0 < r_1 \le r_2 \le r_3 .... \le r_n \le ....; r_n \to \infty)$$

then the exponent of convergence is given by $\frac{1}{\rho_c} = \lim_{n\to\infty} \frac{\log r_n}{\log n}$.

7. State and prove Weirestrass Factorization theorem.

8. Using Hadamard's factorization theorem show that :

$$\frac{\sin \pi\sqrt{z}}{\pi\sqrt{z}} = \prod_{n=1}^{n}\left(1 - \frac{z}{n^2}\right)$$

9. State and prove Montel-Caratheodory theorem.

10. State and prove little Picard's theorem.

Section B

Note : Attempt all the questions.

11. (a) State and prove Riemann maping theorem.

(b) Let (X, d) be a compact metric space and let $\{g_n\}$ be a sequence of continuous function from X into **C** such that $\sum g_n(x)$ converges absolutely and uniformly for x in X. Then the product

$f(x) = \prod_{n=1}^{\infty}(1 + g_n(x))$ converges absolutely

and uniformly for x in X. Also there is an integer n_0 such that $f(x) = 0$ iff $g_n(x) = -1$ for some n, $1 \le n \le n_0$.