

12. Define dual space. Prove that $(l_p^n)^* = l_q^n$ and

$$(l_1^n)^* = l_\infty^n \text{ where :} \quad 12$$

$$l_p^n = \left\{ x = (x_1, x_2, \dots, x_n) : \|x\| = \left(\sum_{i=1}^n |x_i|^p \right)^{1/p} \right\}$$

$$l_1^n = \left\{ x = (x_1, x_2, \dots, x_n) : \|x\| = \sum_{i=1}^n |x_i| \right\}$$

$$l_\infty^n = \left\{ x = (x_1, x_2, \dots, x_n) : \|x\| = \max_{1 \leq i \leq n} |x_i| \right\}$$

Or

If P is a projection on a Banach space B and if M and N are its range and null space, then prove that M and N are closed linear subspace of B such that $B = M \oplus N$. Also state and prove its converse part.

Roll No.

Exam Code : J-19

Subject Code—0366

M. Sc. EXAMINATION

(Fourth Semester)

(Main & Re-appear Batch 2011 Onwards)

MATHEMATICS

MAL-641

Functional Analysis

Time : 3 Hours

Maximum Marks : 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. Prove that a normed linear space X is complete iff every absolutely summable series in X is convergent (summable).
2. State and prove Minkowski's inequality for L^p space.

3. Let M be a closed linear subspace of a normed linear space N and let x_0 be a vector not in M , then there exists a functional F in N^* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$
4. Let X and Y be normed spaces over the field K and $T: X \xrightarrow{\text{onto}} Y$ be a linear operator. Then, T^{-1} exists and is a bounded linear operator iff $\exists$ a constant $K > 0$ such that $\|Tx\|_Y \geq K \|x\|_X, \forall x \in X$.
5. Prove that a linear transformation is closed iff its graph is a closed subspace.
6. Let X and Y be normed spaces and $T: X \rightarrow Y$ be linear operator. Then prove that T is compact iff it maps every bounded sequence $\langle x_n \rangle$ in X onto a sequence $\langle Tx_n \rangle$ in Y which has a convergence.
7. State and prove Schwarz's inequality in an Inner product space.

8. Show that the linear space $C[a, b]$ equipped with the norm given by $\|x\|_\infty = \sup_{t \in [a, b]} |x(t)|, x \in C[a, b]$, is not an inner product space and hence not a Hilbert space.
9. A subspace M of a Hilbert space H is closed in H iff $M = M^{\perp\perp}$.
10. If N_1 and N_2 are normal operators on a Hilbert space H with the property that either commutes with the adjoint of the other, then prove that $N_1 + N_2$ and $N_1 N_2$ are normal.

Section B

Note : Attempt all the questions.

11. State and prove Riesz-Fisher theorem for the completeness of L^p space. 12

Or

State and prove Riesz-representation theorem for bounded linear functional on $C[a, b]$.

13. Let H be a Hilbert space and let $\langle e_i \rangle$ be an orthonormal set in H . Then the following conditions are all equivalent to each other : **11**

- (i) $\langle e_i \rangle$ is complete
- (ii) $x \perp \langle e_i \rangle \Rightarrow x = 0$
- (iii) If x is any arbitrary vector in H , then

$$x = \sum (x, e_i) e_i .$$
- (iv) If x is any arbitrary vector in H , then

$$\|x\|^2 = \sum |(x, e_i)|^2 .$$

Or

Let H be a Hilbert space and let f be an arbitrary functional in H^* . Then there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H .

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