11. Define normal line, normal plane, principal normal, binormal. Also write the equations of normal plane and rectifying plane.

Or

If C be any given curve and C_1 is the locus of the centre of spherical curvature, show that the product of the curvature of these two curves is equal to the product of their torsions.

12. (a) The normal at a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinate planes in A_1, A_2, A_3. Prove that PA_1, PA_2 and PA_3 are in constant ratio.

(b) Show that the envelop of a family of surfaces touches each members of the family at all points of its characteristic.

Or

(a) Show that the curves :

$$(du)^2-\left(u^2+c^2\right)(d\phi)^2=0$$

form an orthogonal system on the surface :

$$x=u\cos\phi,\ y=u\sin\phi,\ z=c\phi$$

Roll No. .... **Exam Code : J-19**

Subject Code—0367-X

M. Sc. EXAMINATION

(Prior 2011 Re-appear)

(Fourth Semester)

MATHEMATICS

MAL-642

Differential Geometry

Time : 3 Hours *Maximum Marks :* 100

Section A

Note : Attempt any *Seven* questions. **7×7=49**

1. Obtain $\left[\vec{b}', \vec{b}'', \vec{b}'''\right]$, where symbols have their usual meanings.

2. Find curvature for the curve :

$$x=a\left(3\theta-\theta^3\right),\ y=3a\theta^2,\ z=a\left(3\theta+\theta^3\right)$$

3. If the tangent and binormal at a point of a curve make angles α and β respectively with a fixed direction, show that :

$$\frac{\sin\alpha\, d\alpha}{\sin\beta d\beta} = -\frac{k}{\tau}$$

4. Prove that, for curves drawn on the surface of a sphere :

$$\frac{\rho}{\sigma} + \frac{d}{ds}(\sigma\rho') = 0$$

where symbols have their usual meaning.

5. Find the equation of the tangent plane and normal to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$.

6. Find the edge of regression of the envelope of the family of planes :

$$x\sin\theta - y\cos\theta + z = a\theta$$

where θ being the parameter.

7. Find the condition that the two directions given by the quadratic :

$$\mathrm{P}du^2 + \mathrm{Q}du dv + \mathrm{R}dv^2 = 0$$

are orthogonal.

8. Prove that the surface of revolution :

$$x = u\cos\phi,\ y = u\sin\phi,\ z = a\log\left\{u + \sqrt{u^2 - a^2}\right\}$$

is a minimal surface.

9. Prove that :

$$\mathrm{H}[\vec{n}, \vec{n}_1, \vec{r}_1] = \mathrm{EM} - \mathrm{FL}$$

10. If K and τ are the curvature and torsion of a geodesic, prove that :

$$\tau^2 = (k - k_a)(k_b - k)$$

where symbols have the usual meanings.

Section B

Note : Attempt all the questions. **3×17=51**

(b) Prove that :

$$T^2 \vec{r}_1 = (FM - EN)\vec{n}_1 + (EM - EL)\vec{n}_2$$

13. Find the principal directions and the principal curvatures on the surface :

$$x = a(u+v),\ y = b(u-v),\ z = uv$$

Also show that the parametric curves on the surface are straight lines.

Or

Obtain the following differential equations of the geodesic on the surface $\vec{r}(u, v)$, where $u(s), v(s)$:

$$\frac{d}{ds}(Eu' + Fv') = \frac{1}{2}\left[E_1 u'^2 + 2F_1 u'v' + G_1 V'^2\right] \text{ and}$$

$$\frac{d}{ds}(Fu' + Gv') = \frac{1}{2}\left[E_2 u'^2 + 2F_2 u'v' + G_2 V'^2\right],$$

where symbols have their usual meanings.

(b) Prove that :

$$T^2 \vec{r}_1 = (FM - EN)\vec{n}_1 + (EM - EL)\vec{n}_2$$

13. Find the principal directions and the principal curvatures on the surface :

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$$\frac{d}{ds}(Fu' + Gv') = \frac{1}{2}\left[E_2 u'^2 + 2F_2 u'v' + G_2 V'^2\right],$$

where symbols have their usual meanings.