

Section B**Note :** Attempt all the questions. **3×17=51**

11. (a) Apply Fourier transform to solve the integral equation :

$$\int_0^{\infty} g(s) \cos(ps) ds = \begin{cases} 1-p, & 0 \leq p \leq 1 \\ 0 & p > 1 \end{cases}$$

- (b) Solve :

$$f(x) = \int_x^b \frac{y(t) dt}{(t^2 - x^2)^{\alpha}}, \quad 0 < \alpha < 1; a < x < b$$

Or

- (a) Obtain the approximate solution of the integral equation :

$$g(s) = s^2 + \int_0^1 \sin(st) g(t) dt$$

by using first two terms of the power series expansion of $\sin(st)$.

- (b) Solve the integral equation :

$$f(x) = \int_x^b \frac{g(t)}{(\cos x - \cos t)^{1/2}} dt; \quad 0 \leq a < x < b \leq \pi$$

J-0369-X**4****Roll No.****Exam Code : J-19****Subject Code—0369-X****M. Sc. EXAMINATION**

(Prior to 2011 Re-appear)

(Fourth Semester)

MATHEMATICS**MAL-644****Integral Equations***Time : 3 Hours**Maximum Marks : 100***Section A****Note :** Attempt any *Seven* questions. **7×7=49**

1. Define Volterra integral equation of first and second kind and obtain an integral equation corresponding to :

$$\frac{d^2 y}{dx^2} + y = e^x \quad y(0) = 0, y'(0) = 1$$

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2. Show that the homogeneous integral equation :

$$y(x) = \lambda \int_0^1 (3x-2)t y(t) dt$$

has no eigenvalues and eigenfunctions.

3. Define resolvent kernel of an integral equation. Using resolvent kernel, solve the following integral equation :

$$g(s) = 1 + \lambda \int_0^1 (1-3st) g(t) dt$$

4. Solve the integral equation :

$$y(x) = 1 + \int_0^1 (1 + e^{x+t}) y(t) dt$$

5. Define integral equation of convolution type. Solve the symmetric integral equation :

$$g(s) = 1 + \lambda \int_0^\pi \cos(s+t) g(t) dt$$

6. Construct Green's function corresponding to the boundary value problem :

$$s^2 \frac{d^2 y}{ds^2} + s \frac{dy}{ds} + (\lambda s^2 - 1) y = 0,$$

$$y(0) = 0, y(1) = 0.$$

7. With the use of Green's function convert the following boundary value problem :

$$y'' + \lambda y = 0; y(0) = 0, y'(1) + v_2 y(1) = 0$$

to a Fredholm integral equation.

8. Apply Laplace transform to solve the integral equation :

$$\int_0^s \frac{g(t)}{(s-t)^{1/2}} dt = 1 + s + s^2$$

9. Show that eigenfunctions of a symmetric kernel corresponding to two distinct eigen-values are orthogonal.

10. Define orthogonal system of functions. Also, discuss the Gram-Schmidt method for construction of an orthogonal set $\{\phi_1, \phi_2, \phi_3, \dots, \phi_k, \dots\}$ from given independent set of functions $\{\psi_1, \psi_2, \psi_3, \dots, \psi_k, \dots\}$.

Or

- (a) Using Hilbert-Schmidt theorem find solution of the integral equation :

$$g(s) = (s+1)^2 + \int_{-1}^1 (st - s^2 t^2) g(t) dt$$

- (b) Using resolvent kernel, find the solution of the integral equation :

$$y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} y(t) dt$$

12. (a) Define Fredholm operator K, its conjugate and show that :

$$(K, \phi, \psi) = (\phi, K \psi)$$

- (b) Using Laplace transform solve the integral equation :

$$f(t) = t + 2 \int_0^t \cos(t-x) f(x) dx$$

Or

- (a) State and prove Hilbert-Schmidt theorem.
(b) Determine the modified Green's function for the boundary value problem :

$$y''(x) = 0; \text{ when } y'(0) = y'(1) = 0$$

13. (a) Apply Fourier transform to solve the integral equation :

$$\int_0^\infty g(s) \cos(ps) ds = \begin{cases} 1-p, & 0 \leq p \leq 1 \\ 0 & p > 1 \end{cases}$$

- (b) Find resolvent kernels for the kernel $K(s, t) = e^{(s+t)}$; with lower limit of the integral $a = 0$ and the upper limit $b = 1$.